How to Estimate the Sigma Level of the Process

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ABSTRACT

Purpose: This paper proposes a procedure for estimating the sigma level of the process through a confidence interval.

M**ethodology/Approach:** The approach is based on a model in which the process has a normal distribution and constant variance, and its mean is shifted to the right or left by 1.5 standard deviations.

Findings: The paper explains a method for creating a random sample along with determining the sample size to estimate the "defects per million opportunities" characteristic through a confidence interval. Based on it, the confidence interval for the "sigma level" of the process is determined.

Research Limitation/implication: We assume a discrete process in which *n* pieces of the product are selected. The proposed procedure assumes that the process is in statistical control.

Originality/Value of paper: Applying the proposed random sampling and estimation procedure can improve process performance evaluations, aiding decision-making for Six Sigma projects.

Category: Research paper

Keywords: Six Sigma; defects per million opportunities; creating a random sample; confidence interval for sigma level

Research Areas: Quality Engineering

1 INTRODUCTION

Six Sigma is a quality assurance and improvement initiative that was introduced at Motorola in the late 1980s. The term Six Sigma has three different meanings, depending on the context. Firstly, it is used to refer to a measure of quality. The Greek letter Sigma is used to measure the variation in a process. Secondly, Six Sigma is a business improvement strategy and a philosophy. Thirdly, it is a problem-solving methodology that aims to identify and eliminate the causes of defects or mistakes in business processes by focusing on critical process outputs in customers' eyes (Antony et al., 2016). Let's examine the last two meanings in more detail. According to Allen (2019), Six Sigma is defined as an organised and systematic problem-solving method for strategic system improvement and new product and service development that relies on statistical methods and the scientific method to dramatically reduce customer-defined error rates and/or improve key output variables. The main reason for the success and popularity of this methodology today is the use of a rigorous methodology to identify and eliminate sources of variability (Stamatis, 2003). Top firms like Toyota and GE have relied on Six Sigma to build their performance cultures (Bloom, 2022).

The Six Sigma approach is project-oriented and focused on the organisation's strategic business goals. The main purpose of a Six Sigma project is to solve a given problem to contribute to the achievement of the organisation's business objectives. The fundamental philosophy is centred around increasing customer satisfaction by eliminating and preventing nonconformities, ultimately leading to increased business profitability. This approach primarily employs statistical tools and should be aligned with risk management plans and other nonconformity prevention activities. Generally, when implementing a Six Sigma project, the principles are adopted that the project is started only after the development of its adequate financial justification and that the practitioners who implement the Six Sigma project can and should benefit from the application of statistical methods without the help of statistical experts (Allen, 2019).

The Six Sigma system improvement activities are closely associated with the DMAIC procedure, which includes five phases: define, measure, analyse, improve, and control. During the define phase, the problem to be addressed is identified and defined. In the measure phase, the current performance of the process to be improved is measured. The analysis phase is focused on identifying the main causes of low performance. The improvement phase involves testing and studying potential solutions to create a robust, improved process. Finally, the control phase ensures that the improved process is maintained by applying a standardised process that can be operated and continuously improved, making the improved performance sustainable over time.

The voice of the customer (VOC) should provide constant feedback throughout the Six Sigma project. In a Six Sigma project context, this can be the project sponsor, an internal or an external customer. The Six Sigma project should always start with identifying the customer's needs and expectations. As the project

progresses, each stage should be checked to ensure that we are meeting the initial customer expectations. The Six Sigma methodology should focus not only on customer satisfaction but also on financial efficiency and safety. To evaluate the financial acceptability of the project, the first step should be to create an accounting model. The project's performance should then be assessed in terms of efficiency for both the customer and the business.

Quantities that can affect the quality of the final product are collectively called critical-to-quality characteristics (CTQCs). These are measurable characteristics of a product or service whose performance standards or tolerance limits must be met to meet VOC requirements.

This paper's main focus is to estimate a process's sigma level. To achieve this, we will begin by estimating the number of defects (nonconformities) per million opportunities DPMO using a confidence interval. We will also propose the sampling method and sample size calculation for this estimation. With the confidence interval for DPMO in hand, we can then determine the corresponding confidence interval for the sigma level of the process. Implementing these procedures should help make better decisions on Six Sigma projects.

2 LITERATURE REVIEW

The primary metric of Six Sigma is the defect per million opportunities DPMO (Le and Duffy, 2023). The term DPMO, its calculation in the sample, and its relationship to the sigma level of a process are explained in Antony et al. (2016), Gitlow et al. (2015), Patel (2016), Bass (2007), Bass and Lawton (2009), and Sproull (2019). Meran et al., (2013) not only explain how DPMO is calculated but also highlight its usage in specific steps of the DMAIC procedure. Furthermore, Basu (2009), Pyzdek (2003), and Pyzdek and Keller (2010) show how DPMO is related to other process performance characteristics.

The DPMO value in the population is estimated using the sample number of defects per million opportunities $\widehat{\mathrm{DPMO}}$. It is necessary to determine the sample size and the sampling method. Bhakri (2023) states only a very general rule that the sample size should be small enough to be well-managed and, at the same time, large enough to capture all the specifics of the process. Furthermore, the websites of some Six Sigma companies1 recommend determining whether the sample is large enough to allow for nonconformities to occur and that determining the sample size should be based on knowledge of the nature of the process. However, the literature above provides no detailed instructions on how to create a sample. It is often not even stated whether the used sample is the result of random or nonrandom sampling. Sometimes it is stated that the obtained results are estimates, indicating the necessity of making a sample by random sampling. According to ISO 22514-3 (2020, p. 2), the methods for measuring machine performance

[1] For example, ISIXSIGMA, Master of Project Academy.

mentioned in the document can also be used during process audits, and the sample size, in this case, should be at least 50. In ISO 13053 – 2 (2011, p. 32), the general procedure for calculating the sample size for a given confidence coefficient of 0.95 and the margin of error in the confidence interval when estimating the proportion and the mean is given. Nonetheless, no more precise procedures exist in the cited literature for creating the sample when estimating DPMO.

3 METHODOLOGY

Let's start by looking at a single critical-to-quality characteristic of a product or a simple process that involves only one step. We will assume it follows a normal distribution. If its value falls within the tolerance limits - LSL (lower specification limit) and USL (upper specification limit), then the product is considered conforming. If the value falls outside of these limits, the product is nonconforming.

When designing the Six Sigma concept, the assumption was made that the process mean is shifted from the target value by 1.5 standard deviations to the right or to the left. To ensure reliable prediction of process performance, the process must be stable (in control), meaning the probability distribution parameters of the monitored characteristic do not change over time. However, in reality, process disturbances may occur that cause the process mean to deviate from the target value. Accumulation of small shifts in the process mean in the long period can lead, in the least favourable case, to a shift in the process mean of 1.5 standard deviations to the right or left of the target value (Bass, 2007). According to ISO 13053 – 1 (2011), 1.5 sigma is an estimate of the process mean shift between the short and long periods.

When using Shewhart control charts2 to monitor a process, it is important to note that they may not detect small shifts in the process that are 1.5 standard deviations or less (Montgomery, 2013). This means that in practice, there can often be situations where the process works with a shift in the mean value by up to 1.5 standard deviations without receiving an out-of-control signal in the control chart. Thus, the process can be considered stable for a certain time when the mean value is shifted by 1.5 standard deviations. Therefore, in the concept of Six Sigma, this kind of process behaviour is modelled. While not completely accurate, this model has proven suitable for evaluating process performance.

Let's notice how the proportion of conforming items can be calculated. Let the characteristic X have a normal distribution with a mean $\mu = \mu_0 + 1.5\sigma$, were μ_0 is the target value of the characteristic in the center of the tolerance band and with a standard deviation σ.

In general, the upper and lower specification limits USL and LSL are USL = $\mu_0 + z\sigma$; LSL = $\mu_0 - z\sigma$, where z is the value of the random variable Z

[2] For more details on Shewhart control charts see Mongomery (2013), Terek and Hrnčiarová (2004).

with a standard normal distribution and determines the number of standard deviations σ, which number indicates the sigma level of quality. For example, for $z = 3$, it is a three sigma level, for $z = 6$, it is a six sigma level, and so on. The probability that the item is conforming is $P(\text{LSL} \leq X \leq \text{USL})$. Assuming that $\mu = \mu_0 + 1{,}5\sigma$, we will standardise USL and LSL:

$$z_{\text{USL}} = \frac{\mu_0 + z\sigma - (\mu_0 + 1.5\sigma)}{\sigma} = z - 1.5$$
$$z_{\text{LSL}} = \frac{\mu_0 - z\sigma - (\mu_0 + 1.5\sigma)}{\sigma} = -z - 1.5 = -(z + 1.5)$$

where z_{USL}, z_{LSL} are the values of the random variable Z with a standard normal distribution. Apparently:

$$P(\text{LSL} \leq X \leq \text{USL}) = P(z_{\text{LSL}} \leq Z \leq z_{\text{USL}})$$

After substituting for z_{LSL} and z_{USL} into the previous relation, and after simple adjustments, we get[3]:

$$P(\text{LSL} \leq X \leq \text{USL}) = \Phi(z - 1{,}5) - 1 + \Phi(z + 1{,}5)$$

where Φ is the distribution function of the standard normal distribution. Its values at various points can be searched using the NORM.S.DIST function in Excel[4]. Then, the probability that the item is nonconforming is:

$$P(X \leq \text{LSL}) + P(X \geq \text{USL}) = 2 - \Phi(z - 1{,}5) - \Phi(z + 1{,}5) \qquad (1)$$

We get the same result for $\mu = \mu_0 - 1.5\sigma$. In Table 1 are the probabilities that the item is nonconforming for $z = 2$; 2.5; 3; 3.5; 4; 4.5; 5; 5.5; 6. The probability that an item is nonconforming can also be understood as the number of nonconforming per item in the population, and after multiplying it by one million, we get the number of nonconforming per million items in the population in the third column. The table can, of course, be expanded as needed.

[3] More details in Terek (2023b).

[4] See the procedure in (Terek 2017b, p. 39 – 40).

Table 1 – Sigma levels

Sigma level z (z score)	**Probability that an item is nonconforming (number of nonconforming per item)**	**Number of nonconforming per million items**
2 σ	0.308770168	308,770.168
2,5 σ	0.158686925	158,686.925
3 σ	0.066810599	66,810.599
3,5 σ	0.022750419	22,750.419
4 σ	0.006209684	6,209.684
4,5 σ	0.001349899	1,349.899
5 σ	0.000232629	232.629
5,5 σ	0.000031671	31.671
6 σ	0.000003398	3.398

Source: Terek (2023b)

3.1 The number of defects per million opportunities and sigma level

One final product (item) is usually characterised by a large number of critical-to-quality characteristics, or its production goes through several steps of the production process. Instead of items, let us now consider the opportunities for defects. Each opportunity either generates or does not generate one defect. Let's consider the probability distribution of some hypothetical quality indicator, the values of which collectively take into account all considered critical-to-quality characteristics. Let the probability distribution of this indicator be the same as when we considered a single critical-to-quality characteristic − normal with the mean μ shifted by 1.5 standard deviations σ to the right or left of the target value μ_0. Then, we can change the names of the second and third columns in Table 1 as follows: the probability that an opportunity generates a defect (also the number of defects per opportunity), the number of opportunities that generate a defect per million opportunities (also the number of defects per million opportunities). Numbers inside table no. 1 will remain unchanged. Thus, the considered probabilistic model and Table 1 can also be used to determine the sigma level based on the number of defects per million opportunities.

The unknown number of defects per million opportunities in the population DPMO will be estimated by the sample number of defects per million opportunities $\widehat{\mathrm{DPMO}}$, whose value in the sample is calculated according to ISO 13053 - 1 (2011) as follows:

$$\widehat{\mathrm{DPMO}} = \frac{c}{n_{\mathrm{units}} \cdot n_{\mathrm{CTQC}}} \cdot 1{,}000{,}000 = \widehat{\mathrm{DPO}} \cdot 1{,}000{,}000 \qquad (2)$$

where $\widehat{\mathrm{DPMO}}$ is the sample number of defects per million opportunities,

c – the number of defects in the sample,

n_{units} – sample size,

n_{CTQC} – the number of critical-to-quality characteristics,

$n^* = n_{\mathrm{units}} \cdot n_{\mathrm{CTQC}}$ – the number of opportunities for defects in the sample,

$\widehat{\mathrm{DPO}}$ – the sample number of defects per opportunity.

Based on the estimated value of DPMO, we can determine the approximate sigma level in Table 1.

In ISO 13053 - 1 (2011) Annex 1 on p. 28 − 29 is Table A.1 − Sigma levels, in which the DPMO values for sigma levels from 0 to 6 are divided by 0.01. It should be noted that the values in the table are not based on the calculation of the probability that the item is nonconforming according to relation (1). They are based on the calculation of the probability that the item is nonconforming with respect to the upper tolerance limit. They count:

$$P(X \geq \mathrm{USL}) = P\,(Z \geq z_{\mathrm{USL}}) = P\,(Z \geq (z - 1{,}5)) = 1 - \Phi(z - 1{,}5)$$

The results in Table A1 differ only slightly from the results obtained by calculating the probability that an item is nonconforming according to relation (1). These small differences will practically not affect the value of the sigma level estimate. If a similar table is not available, an approximate relationship between sigma levels z and DPMO can be used to determine the sigma level (Pillet, 2005, p. 23):

$$z = 0{,}8406 + \sqrt{29{,}37 - 2{,}221 \cdot \ln \mathrm{DPMO}} \qquad (3)$$

4 ESTIMATING DPMO THROUGH CONFIDENCE INTERVAL

We will estimate DPMO and based on the obtained estimate, the corresponding sigma level will be determined. During the practical implementation of the sampling, however, answers to many questions must be found. For example: What is a population? Is it finite or infinite? How and when should random sampling be

implemented? Finally, how does one determine the appropriate sample size? We will try to answer all these questions step by step.

When we want to estimate an unknown value of a parameter in a population, we use a point estimator. A point estimator is a sample statistic, which is a random variable that depends on a set of observations that make up a random sample. The observations are statistically independent and identically distributed, meaning that they have the same probability distribution and are not affected by each other.

Random sampling with replacement of size n from a finite or infinite population or random sampling without replacement from an infinite population is used to ensure that the observations are statistically independent and identically distributed. The population consists of the entire production of the given product. A population in which it is impossible or unrealistic to record every unit in real-time is known to be considered infinite even when in fact, it is finite. A random sample from an infinite population is obtained by selecting n units in a way that satisfies two conditions: each selected unit is from the same population, and each unit is selected independently (Anderson et al., 2020, Terek, 2017a, Terek, 2019, Terek, 2023b). Then, the observations are statistically independent and identically distributed random variables, and the usual methods of statistical inference can be used.

The capability and performance of a process can only be reliably predicted if the process is stable (in statistical control), that is, the parameters of the probability distribution of the process do not change over time. When the process is under statistical control, one random sample of size n can be taken at a time when the process is stable. To ensure the condition that all observations are from the same population is valid, the sample should consist of units that were produced at the same time (or as closely together as possible). Ideally, the consecutive units of production should be taken. The model used assumes the possibility of gradual accumulation of small shifts in the mean value with a constant variance over a longer period up to a shift size of 1.5σ. In a short period, there are only possible small shifts, so the distribution of the population is practically unchanged for a short period. Then, the parameters of the probability distribution of the process are equal or approximately equal, and all observations in the sample are from the same population (probability distribution). The same procedure is followed when estimating short-term capability (machine capability) (ISO 22514-3, 2020). The independence condition should be fulfilled in such a way that the units are produced independently, and thus, the production of each unit can be considered as the implementation of an independent random experiment.

The situation is more complicated if we want to estimate the number of defects per million opportunities when the process is not in statistical control. Mongomery (2013, p. 374) states that if the process is out-of-control, the statistical properties of the process performance indices cannot be determined, and no valid conclusions can be made about their true values in the population. Of course, this also applies to DPMO. If the value of $\widehat{\mathrm{DPMO}}$ was calculated from random sample

data from an out-of-control process, no valid conclusion can be made about the DPMO value in the population and consequently about the sigma level of the process. If we have no information about process stability, estimating process performance can only be an exploratory analysis. The value of $\widehat{\mathrm{DPMO}}$ that was obtained based on the sample data cannot be used to estimate the unknown DPMO value in the population. It is only the value of the descriptive statistic of the process performance in the sample. If, nevertheless, we make any conclusions about the DPMO and consequently about the sigma level in the population based on the calculated value of $\widehat{\mathrm{DPMO}}$, it is only a subjective evaluation.

Alternatively, we may have some clues about the stability of the process. Imagine a situation where the process is not under statistical control but over a longer period, for example, the return rate RR, which is defined as the number of returns or requests to return the product over a certain period, e.g. month, divided by the number of shipped products, on-time delivery OTD, which measures the timeliness of deliveries to customers, the number of problem reports NPR in individual months, or the cost of poor quality COPQ practically remain the same from month to month. It could indicate the stability of the production process at a certain sigma level. However, even in this case, one must be very careful when interpreting the performance characteristics of the process.

Assume that the process is stable. When calculating and interpreting confidence intervals for DPMO and consequently for the sigma level, you can proceed as follows. Each opportunity for a defect generates a defect with probability DPO and does not generate a defect with probability $(1-\mathrm{DPO})$. The probability of DPO, or the number of defects per opportunity in the population, can also be understood as the proportion of opportunities that generate a defect in the population. The proportion DPO can be estimated using $(1-\alpha)\cdot 100\%$ confidence interval:

$$\widehat{\mathrm{DPO}} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}} \leq \mathrm{DPO} \leq \widehat{\mathrm{DPO}} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}} \qquad (4)$$

where

DPO is the proportion of opportunities that generate a defect in the population (also the number of defects per opportunity in the population, also the probability that opportunity generates a defect),

$n^* = n_{\mathrm{units}} \cdot n_{\mathrm{CTQC}}$ – the number of opportunities in the sample,

$z_{1-\frac{\alpha}{2}} - \left(1-\frac{\alpha}{2}\right)\cdot 100\%$ quantile of standard normal distribution,

$d = z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}}$ – the margin of error in the confidence interval (4),

$\widehat{\mathrm{DPO}}$ – the value of the sample proportion of opportunities that generate a defect (also the number of defects per opportunity in the sample)

The relation (4) is recommended to be used when: $n^*\widehat{\mathrm{DPO}} > 5$ and at the same time $n^*\left(1 - \widehat{\mathrm{DPO}}\right) > 5$. Since DPO is also the number of defects per opportunity in the population, multiplying the calculated confidence interval by one million gives us the confidence interval for DPMO – the number of defects per million opportunities in the population. The lower and upper limits of this interval determine the corresponding lower and upper limits of the $(1 - \alpha) \cdot 100\%$ confidence interval for the sigma level.

Let us now note the problem of determining the sample size. We consider a random sample from an infinite population. It is known that n^* can be calculated for the determined confidence coefficient $(1 - \alpha)$ and the margin of error d according to the relation:

$$n^* = \frac{z^2_{1-\frac{\alpha}{2}} \cdot \mathrm{DPO}(1-\mathrm{DPO})}{d^2} \tag{5}$$

We do not know the DPO value. When determining n^* based on the relationship mentioned earlier, it is possible to use the planning value of DPO instead of its unknown value. For example, this planning value can be obtained through a pilot study, which involves the realisation of a preliminary sample. The sample proportion from this sample $\widehat{\mathrm{DPO}}$ can be used as the planning value[5].

Example. Our final product has four critical-to-quality characteristics. The production process is monitored using Shewhart control charts. During the statistical process control, the control charts did not indicate any shift of the process to an out-of-control state. We will determine the required sample size and estimate the sigma level of the manufacturing process using a 95% confidence interval.

A preliminary sample of 30 units was taken, on which 3 defects (opportunities that generated the defects) were discovered. The value of the sample proportion of opportunities that generate a defect $\widehat{\mathrm{DPO}}$ is then

$$\widehat{\mathrm{DPO}} = \frac{3}{30 \cdot 4} = 0.025$$

If, for example, we determine the margin of error $d = 0.01$, after substituting 1.96 for $z_{1-\frac{\alpha}{2}}$, 0.025 for DPO, and 0.01 for d, to (5) we can calculate the required number of opportunities for defects in the sample, n^*. The result is $n^* = 936.39$.

[5] You can explore additional options of obtaining a planning value in Anderson and al. (2020, p. 395).

Therefore, the necessary sample size[6] is $n = 936.39/4 = 234.0975 \approx 235$. Let's say that in the supplementary sample of the size of $235 - 30 = 205$ units, 17 defects were found, so in total, 20 defects were found in the sample of the size 235 units. Then

$$\widehat{\mathrm{DPO}} = \frac{20}{235 \cdot 4} = 0.021$$

and

$$\widehat{\mathrm{DPMO}} = \widehat{\mathrm{DPO}} \cdot 1{,}000{,}000 = 21{,}000$$

According to relation (4), a 95% confidence interval for DPO can be calculated. The resulting interval is [0.01183372; 0.03016628]. If we multiply this interval by one million, we can obtain a 95% confidence interval for DPMO, which is [11,834; 30,166]. This means that at 95% confidence, the number of defects per million opportunities belongs to the interval [11,834; 30,166]; respectively, at 95% confidence, the value of $\widehat{\mathrm{DPMO}} = 21\,000$ will not differ from the unknown true value of DPMO in the population by more than[7] 10,000. By referencing Table A.1. in ISO 13053 - 1 (2011) we can determine the corresponding confidence interval for the sigma level. At 95% confidence, we produce at the sigma level belonging to the interval [3.38; 3.76].

Another condition for determining the sample size should be the fulfilment of the requirement that at least one defect occurs on the random sample units. It does not matter if we find no defect in the sample of the size 100, 1000, or any other size. The model is unusable when $c = 0$. Therefore, to maintain the criterion's discriminating ability, it is necessary to ensure that at least one defect is found in the sample. It is possible that a random sample of the size n will not have any defects. In such cases, it is recommended to continue the random sampling until the first defect appears or repeat the entire random sampling.

Determining the number of critical-to-quality characteristics n_{CTQC} is a crucial factor in calculating of $\widehat{\mathrm{DPO}}$. To determine these characteristics, you need to create a list of potential defects that may bother customers. It is necessary to focus on characteristics that may affect customer satisfaction and are measurable. Adding irrelevant characteristics to the list can produce a lower estimated DPMO value, a

[6] We consider four critical-to-quality characteristics.

[7] Since we entered $d = 0.01$, which is the margin of error in the confidence interval for DPO, $0.01 \cdot 1{,}000{,}000 = 10{,}000$ defects per million opportunities will be the margin of error in the corresponding confidence interval for DPMO.

higher sigma level, and a false impression of better process performance. In general, the DPMO value can be arbitrarily reduced by adding additional critical-to-quality characteristics, so DPMO estimates for more than one characteristic should be treated with great care (Pyzdek and Keller, 2010, p. 171). Unfortunately, there is no clear guidance on how to determine the number of critical-to-quality characteristics.

We strongly believe that if a company regularly evaluates a process using the estimated DPMO, the number of critical-to-quality characteristics does not change, and all the necessary prerequisites for its estimation are met, there can be no serious objections to this characteristic, and we don't have to worry to determine the corresponding sigma level of the process based on the estimated DPMO.

5 CONCLUSION

The paper proposes a procedure for estimating the DPMO through a confidence interval, with an emphasis on the methods of obtaining data and their use in estimating. Based on it, the confidence interval for the sigma level of the process can be determined.

We start from the clarification of the term Six Sigma and from the model for measuring the sigma level of the process. We have presented a method for calculating the sample statistic "sample number of defects per million opportunities" by which we estimate the "number of defects per million opportunities in the population − DPMO". The relationship between DPMO and the sigma level of the process has been described. In the section "Estimating DPMO through confidence interval ", the method of taking a sample and the possibility of determining its size were analysed. When the process is under statistical control, one random sample of size n can be taken at a time when the process is stable. To ensure the condition that all observations are from the same population is valid, the sample should consist of units that were produced at the same time (or as closely together as possible). Ideally, the consecutive units of production should be taken. When estimating the DPMO using a confidence interval, the sample size can be calculated for the specified confidence and margin of error. Another condition should be the fulfilment of the requirement that at least one defect occurs on the randomly selected items. Once we have a confidence interval for the DPMO, we can easily determine the corresponding confidence interval for the sigma level.

When trying to predict the performance of a process with no information about its stability, the analysis is only exploratory. A random sample can be created similarly to that of a stable process, but the value of $\widehat{DPMO}$ obtained from the sample cannot be used as an estimate for the value of the DPMO in the population. It merely represents a value of descriptive statistics of the process performance in the sample. If we nevertheless estimate the DPMO in the population and subsequently the sigma level of the process with thus obtained value $\widehat{DPMO}$, this is only a subjective assessment.

If we only have some clues about the stability of the process, for example, that in an unmonitored process, the RR, NPR, OTD, or COPQ practically does not change over a longer period, this may indicate the stability of the process at a particular sigma level. However, one must be very careful when interpreting DPMO, sigma level, and other process performance characteristics, even in such cases.

Applying the proposed procedures to sampling and DPMO and Sigma level estimation can allow for more accurate and reliable process performance evaluations and lead to better decision-making on Six Sigma projects.

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CONFLICTS OF INTEREST

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