Estimating Some Metrics in Six Sigma Through Confidence Intervals

DOI: 10.12776/qip.v28i1.1972

Milan Terek

Received: 2024-01-09 Accepted: 2024-03-06 Published: 2024-03-31

ABSTRACT

Purpose: This paper suggests a methodology for estimating some Six Sigma metrics using confidence intervals.

Methodology/Approach: This approach assumes that the process follows a normal distribution with a constant variance. The mean of the process is shifted from the target value to the right or left by 1.5 standard deviations. The estimates are based on a random sample of size *n* taken during a time when the process is stable.

Findings: The paper describes how to create confidence intervals for the number of defects per unit, the probability that a unit will be free of defects, and the rolled throughput yield.

Research Limitation/implication: We assume a discrete process in which *n* units of the product are selected during a time when the process is stable.

Originality/Value of paper: By applying the proposed estimation procedures, process performance evaluations can be improved, facilitating decision-making for Six Sigma projects.

Category: Research paper

Keywords: confidence interval; defects per unit; free of defects unit; rolled throughput yield

Research Areas: Quality Engineering

1 INTRODUCTION

The quality principles outlined in the quality management system standards ISO 9000:2015 and ISO 9001:2015 call for a factual approach to decision-making, a process approach to achieving quality, and the practice of continual improvement. Six Sigma methods are powerful tools for top performance in these areas (ISO 13053 – 1, 2011).

The concept of Six Sigma was developed in the 1980s at Motorola as an approach to reducing variation in production processes. More metrics are used in Six Sigma. The purpose of the metrics in a Six Sigma project is to quantify the performance of a process. The primary reason for this is the number of defects per million opportunities (DPMO). This measure is directly related to the sigma level (sigma score) of the process, which is the benchmark used to rank the quality or performance of the process. In Six Sigma, the higher the Sigma level, the better the process output, which translates into fewer errors, lower operating costs, lower risks, improved performance, and better use of resources (Le and Duffy, 2023). Except for the mentioned metrics, the number of defects per unit (DPU) and the rolled throughput yield (RTY) are frequently used. RTY is the probability that a single unit can pass through a series of process steps free of defects (ISO 13053 – 1, 2011, p. 7).

The paper builds on the results presented in Terek (2023b), in which a method for creating a random sample along with determining the sample size to estimate the number of defects per opportunity DPO through a confidence interval is explained. Based on this, the confidence interval for the DPMO and the "sigma level" of the process are determined.

The main focus of this paper is to propose a methodology for estimating some Six Sigma metrics using confidence intervals. We will show that the confidence intervals for DPU and the probability that a unit will be free of defects can be formulated based on the confidence interval for DPO. Furthermore, the confidence interval for RTY can be formulated based on the created random sample. Implementing these procedures makes it possible to improve decision-making on Six Sigma projects.

2 LITERATURE REVIEW

In Pyzdek (2003), and Pyzdek and Keller (2010), DPMO is defined, and for calculation of the RTY, the metric DPMO of the process steps is used. In Bass (2007), and Bass and Lawton (2009), the calculation of DPMO, DPO, DPU, and RTY is described. The Poisson distribution model is used to estimate RTY through DPU, which is the mean of the distribution. RTY is calculated by multiplying the throughput yields of the process steps[1]. In Antony et al. (2016), the DPMO, DPO,

[1] The throughput yield of the step is the proportion of defective items from the production in this step.

and RTY are defined, and their computing is described. Additionally, ISO 13053 – 1, 2011, explains the calculation of the DPMO and RTY.

Gitlow et al. (2015), Patel (2016), and Basu (2009) provide information on the relationships among DPMO, DPO, DPU, and RTY and explain how these metrics are computed. However, they only list the formulas for calculating the metrics in the sample and do not provide detailed instructions on how to create a sample or determine the sample size.

In Terek (2023a), the method for creating a random sample is described, along with an explanation of the relationships among DPMO, DPO, DPU, and RTY. The point estimation of the mentioned metrics is considered. Terek (2023b) explained the method for creating a random sample and determining the sample size needed to estimate “the number of defects per opportunity” metric in the population through a confidence interval.

3 METHODOLOGY

The approach is based on a model which assumes that the process has a normal distribution with constant variance. To ensure reliable prediction of process performance, the process must be stable (in an in-control state), meaning the probability distribution parameters of the monitored characteristic do not change over time (Montgomery, 2013, p. 29). However, process disturbances may occur that cause the process mean to deviate from the target value even when the process is stable. In the least favorable case, accumulating small shifts in the process mean in the long period can lead to a shift in the process mean of 1.5 standard deviations to the right or left of the target value (Bass, 2007). Therefore, the Six Sigma concept was designed in such a way that the process mean is shifted from the target value by 1.5 standard deviations to the right or left.

The population consists of the entire production of the given product. A population in which it is impossible or unrealistic to record every unit in real-time is known to be considered infinite even when, in fact, it is finite. A random sample from an infinite population is obtained by selecting n units in a way that satisfies two conditions: each selected unit is from the same population, and each unit is selected independently (Anderson et al., 2020, p. 324). Then, the observations are statistically independent and identically distributed random variables, and the usual methods of statistical inference can be used.

In Terek (2023b) it is explained that when the process is under statistical control, one random sample of size n can be taken at a time when the process is stable. To ensure the condition that all observations are from the same population is valid, the sample should consist of units that were produced at the same time (or as closely together as possible). Ideally, the consecutive units of production should be taken. The independence condition should be fulfilled in such a way that the units are produced independently, and thus, the production of each unit can be considered as the implementation of an independent random experiment. Then, all observations are mutually independent and of the same distribution.

In the same paper, the confidence interval for DPO is formulated as follows[2]:

$$\widehat{\mathrm{DPO}} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}} \leq \mathrm{DPO} \leq \widehat{\mathrm{DPO}} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}} \qquad (1)$$

where

$\widehat{\mathrm{DPO}} = \frac{c}{n_{\mathrm{units}} \cdot n_{\mathrm{CTQC}}}$ is the value of the sample proportion of opportunities that generate a defect (also the number of defects per opportunity in the sample),

$z_{1-\frac{\alpha}{2}} - \left(1-\frac{\alpha}{2}\right) \cdot 100\%$ quantile of standard normal distribution,

$n^* = n_{\mathrm{units}} \cdot n_{\mathrm{CTQC}}$ – the number of opportunities in the sample,

c – the number of defects in the sample,

n_{units} – sample size,

n_{CTQC} – the number of critical-to-quality characteristics,

DPO – the proportion of opportunities that generate a defect in the population (also, the number of defects per opportunity in the population, also the probability that opportunity generates a defect),

$d = z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{DPO}}\,(1-\widehat{\mathrm{DPO}})}{n^*}}$ – the margin of error in the confidence interval (1).

The relation (1) is recommended to be used when $n^*\widehat{\mathrm{DPO}} > 5$ and, at the same time $n^*\left(1-\widehat{\mathrm{DPO}}\right) > 5$.

4 CONFIDENCE INTERVALS FOR DPU, THE PROBABILITY THAT A UNIT WILL BE FREE OF DEFECTS, AND RTY

The DPU in the population can be estimated with the aid of the sample number of defects per unit $\widehat{\mathrm{DPU}}$, the probability that a unit will be free of defects using the

[2] The value of n^* can be calculated for the determined coefficient $(1-\alpha)$ and the margin of error *d* (see Terek, 2023b).

Poisson probability distribution, and RTY with the aid of the sample rolled throughput yield $\widehat{\text{RTY}}$.

4.1 Confidence intervals for DPU

The confidence interval for DPU will be formulated based on the confidence interval for DPO. The value of the sample number of defects per unit $\widehat{\text{DPU}}$ can be calculated using the following formula:

$$\widehat{\text{DPU}} = \frac{c}{n_{units}} \tag{2}$$

After obtaining the confidence interval for DPO, we can easily calculate the confidence interval for DPU by multiplying it with n_{CTQC}. This is because DPU = DPO · n_{CTQC} and, $\widehat{\text{DPU}}$ = $\widehat{\text{DPO}}$ · n_{CTQC}. Therefore $(1-\alpha)\cdot 100\%$ confidence interval for DPU is:

$$n_{\text{CTQC}}\left(\widehat{\text{DPO}} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\text{DPO}}\,(1-\widehat{\text{DPO}}\,)}{n^*}}\right) \leq \text{DPU} \leq n_{\text{CTQC}}\left(\widehat{\text{DPO}} + \right.$$
$$\left. + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\text{DPO}}\,(1-\widehat{\text{DPO}}\,)}{n^*}}\right) \tag{3}$$

In the following example, we will refer to the study conducted by Terek (2023b).

Example. The study analysed a final product with four critical-to-quality characteristics. The production process was monitored using Shewhart control charts[3]. The confidence coefficient of 0.95 and the margin of error of 0.01 were used to calculate the sample size of 235 units needed for the confidence interval for DPO. During the random sampling, the control charts did not indicate any shift of the process to an out-of-control state. In the obtained random sample, 20 defects were found. Based on this, the 95% confidence interval for DPO was calculated. The resulting interval was [0.01183372; 0.03016628], and the corresponding 95% confidence interval for DPMO was [11,834; 30,166]. Using this interval, the 95% confidence interval for the sigma level was determined as [3.38; 3.76].

We will now calculate the corresponding confidence interval for DPU based on the given example.

After substitution of $\widehat{\text{DPO}} = \frac{20}{235\cdot 4} = 0.021$; $n_{\text{CTQC}} = 4$; $z_{1-\frac{\alpha}{2}} = 1.96$; $n^* = 235 \cdot 4 = 940$ into the relation (3) we get:

$$0.047334879 \leq \text{DPU} \leq 0.120665121 \tag{4}$$

[3] For more about Shewhart control charts see Montgomery (2013), and Terek and Hrnčiarová (2004).

At 95% confidence, the number of defects per unit in the population is between 0.047334879 and 0.120665121, or the number of defects per thousand units in the population is approximately between 47 and 121.

4.2 Confidence intervals for the probability that a unit will be free of defects

The probability that a particular number of defects will occur on a unit could be of interest. The Poisson distribution can be used to find such probabilities. In general, the Poisson distribution can be used to model the number of successes that occur during a given time interval or in a specified area. It is assumed (Miller and Miller, 2014, p. 163):

(1) the numbers of successes occurring in nonoverlapping time intervals or regions are independent,

(2) the probability of a single success occurring in a very short time interval or in a very small region is proportional to the length of the time interval or the size of the region,

(3) the probability of more than one success occurring in such a short time interval or falling in such a small region is negligible.

The probability function of the Poisson distribution is:

$$p_k = P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad \text{for } k = 0, 1, 2, \ldots$$

where

$\lambda > 0$ is the expected value of the number of successes occurring in the given time interval or region,

k − the number of successes occurring in the given time interval or region.

When we consider the occurrence of a defect as a success, we can model the number of defects that occur on a single unit of the product by using the Poisson distribution. Let $k = 0, 1, 2, \ldots$, be the number of defects that occur on one unit. Then, the probability that k defects will occur on one unit is (Bass, 2007, p. 81) (Bass and Lawton, 2009, p. 132):

$$p_k = P(X = k) = \frac{\mathrm{DPU}^k}{k!} e^{-\mathrm{DPU}}$$

The probability that a unit is free of defects (contains $k = 0$ defects) is:

$$p_0 = P(X = 0) = \frac{\mathrm{DPU}^0}{0!} e^{-\mathrm{DPU}} = e^{-\mathrm{DPU}} \tag{5}$$

and the probability that at least one defect occurs on the unit is:

$$1 - p_0 = 1 - e^{-\mathrm{DPU}} \tag{6}$$

We denote the lower limit and the upper limit in the confidence interval for DPU as $\mathrm{DPU_L}$, and $\mathrm{DPU_U}$. We can estimate the mean of the Poisson distribution in the confidence interval for p_0 by the lower and upper limits of the confidence interval for DPU. So $(1 - \alpha) \cdot 100\%$ confidence interval for p_0 is:

$$e^{-\mathrm{DPU_U}} \leq p_0 \leq e^{-\mathrm{DPU_L}} \tag{7}$$

Example‒continued 1. Continuing our example, we determine the probability of a unit being defective or non-defective. We define a unit as non-defective if there are no defects and consider it as defective if there is at least one defect. To estimate this probability, we create a 95% confidence interval for the probability of a unit being non-defective and for the probability of it being defective.

The interval (4) has a lower limit of $\mathrm{DPU_L} = 0.047334879$ and an upper limit of $\mathrm{DPU_U} = 0.120665121$. Using relation (7), we can determine a 95% confidence interval for the probability that a unit in the population is non-defective. This involves calculating the lower limit $p_{0\mathrm{L}}$ and the upper limit $p_{0\mathrm{U}}$:

$$p_{0\mathrm{L}} = e^{-0.120665121} = 0.886330723$$
$$p_{0\mathrm{U}} = e^{-0.047334879} = 0.953767947$$

At 95% confidence, the probability that the unit in the population is non-defective is between 0.886330723 and 0.953767947. Otherwise, at 95% confidence, the proportion of non-defective units in the population is approximately between 88.63% and 95.38%.

To calculate the probability that the unit is defective, we subtract the probability that the unit is non-defective from 1. For instance, in our example, we can determine a 95% confidence interval for the probability that a unit is defective by calculating the lower limit ($1 - p_{0\mathrm{U}}$) and the upper limit ($1 - p_{0\mathrm{L}}$) of this interval:

$$1 - p_{0\mathrm{U}} = 0.046232053$$
$$1 - p_{0\mathrm{L}} = 0.113669277$$

At 95% confidence, the probability that a unit in the population is defective is between 0.046232053 and 0.113669277. Otherwise, at 95% confidence, the proportion of defective units in the population is approximately between 4.62% and 11.37%.

4.3 Confidence intervals for RTY

RTY, also known as FPY (first pass yield, quality rate), is defined as the ratio of completely defect-free units without any rework[4] during the process at the end of a process to the total number of units at the start of the process (Basu, 2009). It is used as a key performance indicator to measure overall process effectiveness. It is necessary to distinguish between RTY in the population and sample rolled throughput yield $\widehat{\mathrm{RTY}}$.

The confidence interval for RTY will be formulated as the confidence interval for a proportion. We will denote the number of reworked or scrapped units in the sample as c^*. To calculate the sample rolled throughput yield $\widehat{\mathrm{RTY}}$, we can use the formula given below:

$$\widehat{\mathrm{RTY}} = \frac{n_{units} - c^*}{n_{units}} \quad (8)$$

The sample rolled throughput yield can be understood as the proportion of defect-free units in a sample. The unit in the population is defect-free with probability RTY, and it is defective with probability $(1 - \mathrm{RTY})$. The probability of RTY can also be interpreted as the proportion of defect-free units in the population. The proportion RTY can be estimated using $(1 - \alpha) \cdot 100\%$ confidence interval:

$$\widehat{\mathrm{RTY}} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{RTY}}\,(1-\widehat{\mathrm{RTY}})}{n_{units}}} \leq \mathrm{RTY} \leq \widehat{\mathrm{RTY}} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\widehat{\mathrm{RTY}}\,(1-\widehat{\mathrm{RTY}})}{n_{units}}} \quad (9)$$

Example – continued 2. Assume that 20 defects were found across 17 units during the process, where some defects were fixable and others were not. As a result, a total of $c^* = 17$ units were either scrapped or repaired. We calculate a 95% confidence interval for the RTY.

After substituting 235 for n_{units}, and 17 for c^* in relation (8), we get:

$$\widehat{\mathrm{RTY}} = 0.927659574$$

After substituting the value of 0.927659574 for $\widehat{\mathrm{RTY}}$, 1.96 for $z_{1-\frac{\alpha}{2}}$, and 235 for n_{units} in relation (9), we can calculate a 95% confidence interval for RTY:

$$0.894538304 \leq \mathrm{RTY} \leq 0.960780844$$

At 95% confidence, the rolled throughput yield is approximately between 89.45% and 96.08%. Otherwise, at 95% confidence, the proportion of non-defective units in the population is approximately between 89.45% and 96.08%, or the proportion

[4] That means without being rerun, retested, returned, or diverted into an offline repair area.

of units passing through a series of process steps free of defects is approximately between 89.45% and 96.08%.

5 CONCLUSION

The purpose of the paper is to propose the procedures for estimating some metrics in Six Sigma through confidence intervals. A method for estimating process performance characteristics such as DPU, p_0, and RTY are described. The methodology builds on the results presented in Terek (2023b), in which a method for creating a random sample and determining the sample size to estimate the number of defects per opportunity DPO through a confidence interval is offered. Based on this, the confidence interval for the DPMO and the "sigma level" of the process are determined. In this paper, the confidence interval for DPU is formulated based on the confidence interval for DPO. Additionally, based on the confidence interval for DPU, the confidence interval for p_0 is formulated. Besides, based on the created random sample, the confidence interval for RTY is formulated.

We have described a method for calculating the confidence interval $[p_{0L}; p_{0U}]$ for the probability p_0, that no defects occur on a unit. Additionally, we have developed a confidence interval for RTY. According to ISO 13053 – 1 (2011, p. 7), RTY is defined as the probability that a single unit can pass through a series of process steps free of defects. This means that RTY is essentially the same as the probability of no defects occurring on a unit. The main difference between the confidence interval for p_0 and that for RTY, is that the former is based on a theoretical model (Poisson distribution) of the occurrence of a certain number of defects on units, while the latter relies on information obtained from sampling about the proportion of defective units in the sample. In our example, the confidence intervals for p_0, and RTY differ very little.

All procedures are based on the established Six Sigma concept that the mean of a normally distributed process with constant variance is shifted by 1.5 standard deviations to the right or to the left of the target value and on a random sample of size *n* taken in such a way that the consecutive units of production were taken, at a time when the process was stable.

The proposed methods can enhance the accuracy of process performance estimation, leading to better decision-making for Six Sigma projects. They can be very useful, particularly in the define phase of the DMAIC methodology. For instance, consider the results obtained from a random sample of size 235 in our example. At 95% confidence, the number of defects per million opportunities is between 11,834 and 30,166, the sigma level of the process is between 3.38 and 3.76 (Terek, 2023b), the number of defects per thousand units is approximately between 47 and 121, and the proportion of units passing through a series of process steps free of defects is approximately between 89.45% and 96.08%. Armed with such valuable information, we can make better decisions regarding our Six Sigma projects.

ACKNOWLEDGEMENTS

The author thanks the Vysoká škola manažmentu/City University of Seattle for its all-around support of his research activity.

REFERENCES

Anderson, D. R., Sweeney, D.J., Williams, T. A., Camm, J. D., Cochran, J. J., Fry, M. J. and Ohlmann, J. W., 2020. *Statistics for Business and Economics. 14e Edition.* Boston: Cengage Learning, Inc.

Antony, J., Vinodh, S. and Gijo, E.V., 2016. *Lean Six Sigma for Small and Medium Sized Enterprises. A Practical Guide.* Boca Raton: Taylor & Francis Group.

Bass, I., 2007. *Six Sigma Statistics with Excel and Minitab*, New York: McGraw-Hill.

Bass, I., and Lawton, B., 2009. *Lean Six Sigma Using SigmaXL and Minitab*. New York: McGraw-Hill.

Basu, R., 2009. *Implementing Six Sigma and Lean: A Practical Guide to Tools and Techniques.* Oxford: Elsevier Ltd.

Gitlow, H.S., Melnyck, R.J. and Levine, D. M., 2015. *A Guide to Six Sigma and Process Improvement for Practitioners and Students. Foundations, DMAIC, Tools, Cases, and Certification. Second Edition*. Old Tappan: Pearson Education, Inc.

ISO 13053 – 1, 2011. *Quantitative methods in process improvement − Six Sigma − Part 1: DMAIC methodology*. Geneva: ISO copyright office.

Le, H., Duffy, G., 2023. *Human-Centered Lean Six Sigma. Creating a Culture of Integrated Operational Excellence*. New York: Routledge.

Miller, I., Miller, M., 2014. *John E. Freund's Mathematical Statistics with Applications. Eighth Edition.* Harlow: Pearson Education Limited.

Montgomery, D.C., 2013. *Introduction to Statistical Quality Control. Seventh edition.* Hoboken: J. Wiley and Sons.

Patel, S., 2016. *The Tactical Guide to Six Sigma Implementation*. Boca Raton: Taylor & Francis Group.

Pyzdek, T., 2003. *The Six Sigma Handbook. Revised and Expanded. A Complete Guide for Green Belts, Black Belts, and Managers at All Levels*. New York: McGraw-Hill.

Pyzdek, T. and Keller, P.A., 2010. *The Six Sigma handbook. A Complete Guide for Green Belts, Black Belts, and Managers at All Levels. Third Edition*. New York: McGraw-Hill Companies, Inc.

Terek, M. and Hrnčiarová, Ľ., 2004. *Štatistické riadenie kvality*. Bratislava: Iura Edition.

Terek, M., 2023a. Charakteristiky výkonnosti procesu v metodológii Six Sigma. *Slovenská štatistika a demografia*, 33(4), pp. 20 – 40. Avalaible at: https://ssad.statistics.sk/SSaD/wp-content/files/4_2023/4_2023_SSaD_clanok_2_Terek.pdf.

Terek, M., 2023b: How to Estimate the Sigma Level of the Process. Quality Innovation Prosperity 27(3). https://doi.org/10.12776/qip.v27i3.1947.

ABOUT THE AUTHOR

Milan Terek[0000-0001-5638-9287] (M.T.) – Professor, Vysoká škola manažmentu/City University of Seattle, Slovak Republic, e-mail: mterek@vsm.sk; milan.terek1@gmail.com.

CONFLICTS OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.