Dynamic Robust Parameter Design Using Response Surface Methodology based on Generalized Linear Model

DOI: 10.12776/qip.v28i2.2021

Kosuke Oyama, Masato Ohkubo, Yasushi Nagata

Received: 2024-06-05 Accepted: 2024-06-25 Published 2024-07-31

ABSTRACT

Purpose: When designing an input-output system susceptible to noise, engineers assume a functional relation between the input and the output. The Taguchi method, which uses a dynamic, robust parameter design (RPD) to evaluate the robustness of the input-output relation against noise, is employed. This study aims to address extending the scope of use of a dynamic RPD.

Methodology/Approach: A target system in a typical dynamic RPD can be interpreted as one in which the relation between the input and the output is a linear model, and the output error follows a normal distribution. However, an actual system often does not conform to this premise. Therefore, we propose a new analysis approach that can realize a more flexible system design by applying a response surface methodology (RSM) based on a generalized linear model (GLM) to dynamic RPD.

Findings: The results demonstrate that 1) a robust solution can be obtained using the proposed method even for a typical dynamic RPD system or an actual system, and 2) the target function can be evaluated using an adjustment parameter.

Research Limitation/implication: Further analysis is required to determine which factor(s) in the estimated process model largely contribute(s) to changes in the adjustment parameter.

Originality/Value of paper: The applicability of typical dynamic RPD is limited. Hence, this study's analytical process provides engineers with greater design flexibility and deeper insights into dynamic systems across various contexts.

Category: Research paper

Keywords: robust parameter design; dynamic system; generalized linear model; response surface methodology; Taguchi method

1 INTRODUCTION

When designing an input-output system, engineers assume a functional relation between the input and the output (Tatebayashi, 2004). However, in practice, this functional relation does not consistently hold because of noise, which results in critical quality defects. Therefore, the Taguchi method uses a dynamic, robust parameter design (RPD), an indispensable design method, for evaluating the robustness of the input-output relation against noise. The method is centred around reducing variations in a process, and it accounts for noise at the design stage.

In a typical dynamic RPD, the relation between the input M and output y of a system is assumed to be linear, while the output error ε is assumed to follow a normal distribution and possess equal variability. Based on Nagata's (2009) explanation, this assumption can be defined using the following additive model:

$$\begin{aligned} y &= \beta M + \varepsilon, \\ \varepsilon &\sim N(0, \sigma^2). \end{aligned} \tag{1}$$

Hence, methods derived from the RPD (e.g., Kawamura and Takahashi, 2013) also assume that the data follows these assumptions. However, actual systems often do not conform to these assumptions. For example, Mikami and Yano (2004) employed a typical dynamic signal-to-noise ratio analysis with the number of thermotolerant bacteria as the output and the incubation time as the input. However, the number of bacteria per unit of time generally followed a Poisson distribution. Moreover, analyses such as the growth rate of bean sprouts (Yoshino, 1995) assume a growth curve where the relation between the input and output of the system is nonlinear.

To realize a more flexible system design, we focus on a generalized linear model (GLM), a nonlinear model with a nonlinear input-output relation. It is useful for analyzing experimental data because it can handle output errors that follow a non-normal distribution with unequal variability (Lee and Nelder, 1998). If the typical dynamic RPD is reconsidered in the GLM context, the linear predictor then becomes βM, the link function is absent, and the error structure is normally distributed.

Therefore, we propose a new analytical approach that can realize a more flexible system design by applying response surface methodology (RSM) based on GLM to dynamic RPD. The formulation of the proposed method is based on the framework by Myers et al. (2005), in which an approach to RPD was developed to verify its usefulness. In this study, we demonstrated that our proposed method can enable engineers to attain greater design freedom and gain insights into the experimental data of dynamic systems with various backgrounds.

The remainder of this paper is organized as follows. Section 2 explains the approach RPD proposed by Myers et al. (2005). Section 3 proposes a new approach to RPD for dynamic systems. Section 4 analyzes actual data to verify the performance of the proposed method. Section 5 presents the verification results. Section 6 summarizes the study and provides future recommendations.

2 STATIC RPD USING RSM BASED ON GLM

Myers et al. (2005) proposed an approach for static RPD using RSM based on GLM for a static system—a system with fixed inputs. The method aimed to design the system to ensure the output is constantly maintained at the target value. They considered the conditional population mean μ_{ij} of response y_{ij}, as follows:

$$g(\mu_{ij}) = \beta_0 + \mathbf{x}_i^T\boldsymbol{\beta} + \mathbf{x}_i^T\mathbf{B}\mathbf{x}_i + \mathbf{z}_j^T\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda}\mathbf{z}_j,$$
$$\mathbf{x}_i = \begin{pmatrix} x_{i1} \\ \vdots \\ x_{ip} \end{pmatrix}, \mathbf{z}_j = \begin{pmatrix} z_{j1} \\ \vdots \\ z_{jq} \end{pmatrix}, \mathbf{z}_j \sim N(0, \sigma_z^2\mathbf{I}_q). \tag{2}$$

Provided the control factors $\mathbf{x}_i$ and noise factors $\mathbf{z}_j$ use the GLM framework, $\mu = \eta$ and $\eta = \omega(\mathbf{x}, \mathbf{z})$ is the linear predictor. The error structure is chosen to fit the data and the link function $g(\cdot)$ is chosen accordingly. The index $i = 1,2, \dots, I$ denotes the combination of control factors, and $j = 1,2, \dots, J$ denotes the combination of noise factors. β_0 is the intercept parameter. The p-dimensional vector $\boldsymbol{\beta}$ denotes the vector of coefficients for the control factors, while the q-dimensional vector $\boldsymbol{\gamma}$ represents the vector of coefficients for the noise factors. The $p \times q$ matrix $\boldsymbol{\Lambda}$ denotes the matrix of control-by-noise interaction coefficients, and the $p \times p$ matrix $\mathbf{B}$ represents the matrix of second-order effect coefficients of the control factors. Each level of the noise factor follows a normal distribution $N(0, \sigma_{\mathbf{z}}^2)$.

Subsequently, the process mean $E_{\mathbf{z}}(\mu_{ij})$ and process variance $Var(y_{ij})$ were derived. Using the target value T of the response, the following evaluation function was defined as

$$MSE = \left(E_{\mathbf{z}}(\mu_{ij}) - T\right)^2 + Var(y_{ij}). \tag{3}$$

In the RPD proposed by Myers et al. (2005), the design solution is a combination of the control factors that minimize the evaluation function.

3 DYNAMIC RPD USING RSM BASED ON GLM

3.1 Proposal of a new method

In this section, we formulate an approach for dynamic systems based on Section 2. This method comprises four steps. In step 1, the RSM is estimated using a GLM or double-generalized linear model (DGLM). The GLM was proposed by Nelder and Wedderburn (1972) and holds the dispersion parameter as a constant. The

DGLM, an extension of the GLM proposed by Smyth (1989), also models the dispersion parameter by enabling additional flexible modelling.

We consider the conditional population mean μ_{ijk} of the response y_{ijk}, as follows:

$$g(\mu_{ijk}) = \beta_0 + \mathbf{x}_i^T\boldsymbol{\beta} + \mathbf{z}_j^T\boldsymbol{\gamma} + \mathbf{m}_k^T\boldsymbol{\delta} + \mathbf{x}_i^T\boldsymbol{\Lambda}\mathbf{z}_j + \mathbf{x}_i^T\boldsymbol{\Pi}\mathbf{m}_k + \mathbf{z}_j^T\boldsymbol{\Omega}\mathbf{m}_k,$$
$$\mathbf{x}_i = \begin{pmatrix} x_{i1} \\ \vdots \\ x_{ip} \end{pmatrix}, \mathbf{z}_j = \begin{pmatrix} z_{j1} \\ \vdots \\ z_{jq} \end{pmatrix}, \mathbf{m}_k = \begin{pmatrix} f_1(M_k) \\ \vdots \\ f_n(M_k) \end{pmatrix}, \mathbf{z}_j \sim N(0, \sigma_{\mathbf{z}}^2 \mathbf{I}_q). \tag{4}$$

Given that $\mathbf{x}_i$, $\mathbf{z}_j$, and $\mathbf{m}_k$ use the GLM or DGLM framework, $\mu = \eta$ and $\eta = \omega(\mathbf{x}, \mathbf{z}, M)$ is the linear predictor. The error structure is chosen to fit the data and the link function $g(\cdot)$ is chosen accordingly. The index $i = 1,2,\dots,I$ denotes the combination of control factors; $j = 1,2,\dots,J$ represents the combination of noise factors; and $k = 1,2,\dots,K$ denotes the level of the signal factor. β_0 is the intercept parameter, $\mathbf{x}_i$ denotes the control factors, $\mathbf{z}_j$ represents the noise factors and $\mathbf{m}_k$ denotes the functions of the signal factor. The p-dimensional vector $\boldsymbol{\beta}$ signifies the vector of coefficients for the control factors, the q-dimensional vector $\boldsymbol{\gamma}$ denotes that for the noise factors, and the n-dimensional vector $\boldsymbol{\delta}$ denotes that for functions of the signal factor. The $p \times q$ matrix $\boldsymbol{\Lambda}$ represents the matrix of control-by-noise interaction coefficients, the $p \times n$ matrix $\boldsymbol{\Pi}$ is the matrix of the control-by-function of the signal interaction coefficients, and the $q \times n$ matrix $\boldsymbol{\Omega}$ represents the matrix of the noise-by-function of the signal interaction coefficients. Each level of the noise factor follows a normal distribution $N(0, \sigma_{\mathbf{z}}^2)$.

In step 2, we derive the process mean $E_{\mathbf{z}}(\mu_{ijk})$. First, $E_{\mathbf{z}}(\mu_{ijk})$ is obtained by taking the expected value for the noise factors as follows:

$$\begin{aligned} E_{\mathbf{z}}(\mu_{ijk}) = E_{\mathbf{z}}\big(g^{-1}\big[\beta_0 + \mathbf{x}_i^T\boldsymbol{\beta} + \mathbf{z}_j^T\boldsymbol{\gamma} + \mathbf{m}_k^T\boldsymbol{\delta} + \mathbf{x}_i^T\boldsymbol{\Lambda}\mathbf{z}_j + \mathbf{x}_i^T\boldsymbol{\Pi}\mathbf{m}_k \\ + \mathbf{z}_j^T\boldsymbol{\Omega}\mathbf{m}_k\big]\big) = E_{\mathbf{z}}(q[\eta]), \end{aligned} \tag{5}$$

where q is the inverse of the link function g.

Second, using a second-order Taylor series approximation around the mean of the linear predictor $\eta_0 = E_{\mathbf{z}}(\eta)$, the process mean is given as:

$$\begin{aligned} E_{\mathbf{z}}(\mu_{ijk}) &\approx E_{\mathbf{z}}\left(q[\eta_0] + q'[\eta_0](\eta - \eta_0) + \frac{1}{2}q''[\eta_0](\eta - \eta_0)^2\right) \\ &= q[\eta_0] + \frac{1}{2}q''[\eta_0]Var_{\mathbf{z}}(\eta), \end{aligned} \tag{6}$$
$$q'[\eta_0] = \left[\frac{\partial \mu_{ijk}}{\partial \eta}\right]_{\eta=\eta_0}, q''[\eta_0] = \left[\frac{\partial^2 \mu_{ijk}}{\partial \eta^2}\right]_{\eta=\eta_0}.$$

The variance of the linear predictor $Var_{\mathbf{z}}(\eta)$ is defined as

$$Var_{\mathbf{z}}(\eta) = (\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})^T. \tag{7}$$

In step 3, we derive the process variance $Var(y_{ijk})$ using Lee and Nelder's (2003) framework. $Var(y_{ijk})$ is given as:

$$Var(y_{ijk}) = Var_{\mathbf{z}}[E(y_{ijk}|\mathbf{z}_j)] + E_{\mathbf{z}}[\phi_{ijk}V(\mu_{ijk})]. \tag{8}$$

The first term shows the variation due to noise factors, while the second term expresses the variation independent of the noise factors. ϕ_{ijk} (dispersion parameter) represents the variation in population mean μ_{ijk}, and the variance in independent distribution, indicating that it is unaffected by the increase or decrease in the process mean. For simplicity, we consider the dispersion parameter as ϕ_{ik}, which remains constant regardless of noise factors. $V(\mu_{ijk})$ denotes the change in variance relative to the population mean μ_{ijk} of the distribution. This indicates that the variation is affected by the increase or decrease in the process mean.

When estimating the RSM with GLM, ϕ_{ik} is fixed to a constant. When the RSM is estimated using DGLM, we consider the dispersion parameter for the conditional population mean ϕ_{ik} of the response d_{ik}, as follows:

$$\begin{gathered} h(\phi_{ik}) = \beta_0 + \mathbf{x}_i^T\boldsymbol{\beta} + \mathbf{m}_k^T\boldsymbol{\delta} + \mathbf{x}_i^T\boldsymbol{\Pi}\mathbf{m}_k, \\ \mathbf{x}_i = \begin{pmatrix} x_{i1} \\ \vdots \\ x_{ip} \end{pmatrix}, \mathbf{m}_k = \begin{pmatrix} f_1(M_k) \\ \vdots \\ f_n(M_k) \end{pmatrix}, \end{gathered} \tag{9}$$

given $\mathbf{x}_i$ and $\mathbf{m}_k$. The index $i = 1,2,\dots,I$ denotes the combination of control factors, and $k = 1,2,\dots,K$ represents the level of the signal factor. Based on Smyth and Verbyla's (1999) explanation, d_{ik} refers to $d(y,\mu)$ obtained by transforming the exponential family distribution as shown in

$$f(y;\mu,\phi) = b(y,\phi)\exp\left\{-\frac{1}{2\phi}d(y,\mu)\right\}. \tag{10}$$

d is a distance measure between y and μ. β_0 is the intercept parameter, $\mathbf{x}_i$ denotes the control factor and $\mathbf{m}_k$ represents the function of the signal factor. The p-dimensional vector $\boldsymbol{\beta}$ represents the vector of coefficients for the control factors while the n-dimensional vector $\boldsymbol{\delta}$ denotes that for functions of the signal factor. The $p \times n$ matrix $\boldsymbol{\Pi}$ denotes the matrix of the control-by-function of the signal interaction coefficients, and the link function is $h(\cdot)$. Notably, the variables, vectors, and matrices of the coefficient that constitute ϕ_{ik} differ from those of μ_{ijk}. We adopt the estimation method proposed by Smyth and Verbyla (1999).

First, we derive the first term of the process variance, represented in

$$\begin{aligned} Var_{\mathbf{z}}&[E(y_{ijk}|\mathbf{z}_j)] \\ &= Var_{\mathbf{z}}[q(\beta_0 + \mathbf{x}_i^T\boldsymbol{\beta} + \mathbf{z}_j^T\boldsymbol{\gamma} + \mathbf{m}_k^T\boldsymbol{\delta} + \mathbf{x}_i^T\boldsymbol{\Lambda}\mathbf{z}_j + \mathbf{x}_i^T\boldsymbol{\Pi}\mathbf{m}_k \\ &+ \mathbf{z}_j^T\boldsymbol{\Omega}\mathbf{m}_k)]. \end{aligned} \tag{11}$$

This equation can be approximated as:

$$Var_{\mathbf{z}}[\mu_{ijk}] \approx \left[\frac{\partial \mu_{ijk}}{\partial \eta}\right]_{\eta=\eta_0} Var_{\mathbf{z}}[\eta] \left[\frac{\partial \mu_{ijk}}{\partial \eta}\right]_{\eta=\eta_0}, \tag{12}$$

using the delta method. The latter can be represented as:

$$\begin{aligned} Var_{\mathbf{z}}[\mu_{ijk}] \approx {} & [V(\mu_{ijk})]^2_{\eta=\eta_0} \Psi_{\eta=\eta_0}(\boldsymbol{\gamma} + \mathbf{x}_i^T \boldsymbol{\Lambda} \\ & + \mathbf{m}_k^T \boldsymbol{\Omega}) Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T \boldsymbol{\Lambda} + \mathbf{m}_k^T \boldsymbol{\Omega})^T \Psi_{\eta=\eta_0}, \end{aligned} \tag{13}$$

using the chain rule. $\Psi = \partial\theta / \partial\eta$ and θ are location parameters.

Second, we derive the second term of the process variance using a second-order Taylor series approximation around η_0, given as:

$$E_{\mathbf{z}}[\phi_{ik} V(\mu_{ijk})] \approx \phi_{ik} \left\{ [V(\mu_{ijk})]_{\eta=\eta_0} + \frac{1}{2}\left[\frac{\partial^2 V(\mu_{ijk})}{\partial \eta^2}\right]_{\eta=\eta_0} Var_{\mathbf{z}}[\eta] \right\}. \tag{14}$$

Finally, $Var(y_{ijk})$ is defined as

$$\begin{aligned} Var(y_{ijk}) = {} & \left[\frac{\partial \mu_{ijk}}{\partial \eta}\right]_{\eta=\eta_0} Var_{\mathbf{z}}[\eta] \left[\frac{\partial \mu_{ijk}}{\partial \eta}\right]_{\eta=\eta_0} \\ & + \phi_{ik} \left\{ [V(\mu_{ijk})]_{\eta=\eta_0} + \frac{1}{2}\left[\frac{\partial^2 V(\mu_{ijk})}{\partial \eta^2}\right]_{\eta=\eta_0} Var_{\mathbf{z}}[\eta] \right\}. \end{aligned} \tag{15}$$

In step 4, the parameters are optimized. Hence, we define the necessary evaluation function as the adjusted sum of square errors $(adjusted\ SSE)$ for optimization, as follows:

$$adjusted\ SSE = \sum_{M \in \mathrm{M}} \left[\{t - \kappa E_{\mathbf{z}}(\mu_{ijk})\}^2 + Var(y_{ijk}) \right]. \tag{16}$$

M is the set of signal-factor levels. t denotes the target function $q(\beta_{0*} + \boldsymbol{\beta}_*' m)$ and κ represents the adjustment parameter $(0 \leq \kappa)$. The first term measures the deviation between t and $E_{\mathbf{z}}(\mu)$ while the second measures the process variation. κ is used to make adjustments between the target function and the process mean. Hence, the κ^2 scale of variance is omitted. This adjustment allows for the process mean to approach the target function, which is the design concept. If the target function is less than the estimated process mean, then κ is less than 1; otherwise, κ is greater than 1. Thus, the target function can be evaluated.

Next, we explain the optimization method. First, we minimize the $adjusted\ SSE$ with an adjustment parameter of one to derive the design solution. We use the design solution and $\kappa = 1$ to calculate the first term of the $adjusted\ SSE$, which is the evaluation value. Second, we derive the design solution by generating the adjustment parameter and minimizing the $adjusted\ SSE$ to which it is assigned. We use the design solution and the generated adjustment parameter to calculate the first term of the $adjusted\ SSE$. If the first term is less than the evaluation value, the generated adjustment parameter is adopted and the evaluation value is updated.

Otherwise, a new adjustment parameter is generated. This series of processes is repeated until the adjustment parameter yielding the lowest evaluation value is derived. Finally, we find the combination of control factors that minimize the $adjusted\ SSE$ substitution using the adjustment parameter identified in the previous step. The $adjusted\ SSE$ is minimized by determining an adjustment parameter using the simulated annealing method and deriving the design solution using the quasi-Newton method.

The typical dynamic RPD is a two-stage design that first decreases variance and subsequently enables the mean to approach the target function (e.g., dynamic signal-to-noise ratio analysis). Contrastingly, our method first draws the mean closer to the target function and subsequently decreases the variance. This departure arises because the system under consideration entails a tradeoff between mean and variance, a complexity beyond the scope of conventional two-stage design.

3.2 Related research

Kume and Nagata (2013) designed the parameters by defining and minimizing the following equation:

$$WSSE = w \sum_{M \in \mathrm{M}} \left\{ E_{\mathbf{z}}\left(\mu_{ijk}\right) - t \right\}^2 + (1 - w) \sum_{M \in \mathrm{M}} Var\left(y_{ijk}\right), \tag{17}$$

where w denotes an arbitrary weight to balance the first and second terms $(0 \leq w \leq 1)$. If $E_{\mathbf{z}}\left(\mu_{ijk}\right)$ needs to be adjusted to t, w should be set to a higher value; otherwise, it should be set to a lower value. However, a robust solution cannot be obtained without setting an appropriate target function for the estimate process model. This is particularly true when the difference between the process mean and target function is significant. Additionally, the arbitrary nature of the decision of w presents another challenge.

4 VALIDATION OF METHODS USING ACTUAL DATA

4.1 Overview of data

In this section, we verify the design solution using $adjusted\ SSE$ and actual data. The experimental data of the high-speed response valve (also known as a high-speed on/off solenoid valve) employed by Enkawa and Miyakawa (1992) is used along with dynamic systems. In a high-speed response valve, the input-output relation for the generic function is given by the following equation:

$$\text{flow rate} = \text{constant} \times \text{duty ratio} \times \sqrt{\text{pressure difference}}. \tag{18}$$

The input (signal factor) is the square-root transformation of the pressure difference, the output (response y) represents the flow rate, and the duty ratio denotes the ratio of the time the valve is on to the times it is on and off.

Table 1 – Experimental data for the high-speed response valve (Enkawa and Miyakawa, 1992)

	A	B		C			D	S_1		S_2		S_3	
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	G_1	G_2	G_1	G_1	G_2	G_1
1	1	1	1	1	1	1	1	15	33	31	52	58	78
2	1	1	1	2	2	2	2	38	46	69	75	111	126
3	1	2	2	1	1	2	2	22	28	48	51	52	93
4	1	2	2	2	2	1	1	11	38	26	77	95	135
5	2	1	2	1	2	1	2	18	19	42	48	65	71
6	2	1	2	2	1	2	1	39	65	65	98	82	103
7	2	2	1	1	2	2	1	5	45	38	78	38	86
8	2	2	1	2	1	1	2	21	58	59	113	101	121

In this experiment, four control factors are assigned to an inner L_8 orthogonal array. A is the stroke, B is the spring-attachment load, C is the pressure balance, and D is the oil passage area. The noise factor G and signal factor S are arranged in a one-by-one outer array. In the following, the notation of the noise factor is Z, and that of the signal factor is M. The noise factor is the input voltage. The first level of the control factor and noise factor is 1, and the second level is -1. The signal factor is 4 for the first level, 8 for the second level, and 12 for the third level.

These data follow the typical dynamic RPD assumptions. Therefore, it is ideal that the design solution using $adjusted\ SSE$ is consistent with their conclusions—the good design solution is to set $(B, D) = (1, -1)$ to reduce variation in the input-output relation and C to adjust the output.

4.2 Data analysis

First, we estimate the response surface using μ of the link function and DGLM in the normal distribution as follows:

$$\widehat{\mu_{ijk}} = (7.42 - 1.78C - 0.10D)M + (-10.92 + 4.50B - 4.23D)Z,$$
$$\log(\widehat{\phi_{ik}}) = 4.25 - 0.73B - 0.92C. \tag{19}$$

Nair (1992) demonstrated that when the y_{ijk} error structure is normally distributed, the dispersion parameter model follows a gamma distribution with $\log(\mu)$ of the link function. Therefore, $\log(\mu)$ is used as the link function in the dispersion parameter model, which assumes a gamma distribution. Our analysis uses the mean square error calculated using leave-one-out cross-validation. Because control factor A was not selected, setting the level is unnecessary.

In the population mean model, the control factors B and D interact with the noise factors and the control factors B and C are selected in the dispersion parameter model. Therefore, a tradeoff occurs because the control factors decrease the variation while simultaneously adjusting to the target function.

We derive ϕ, $V(\mu)$, and canonical link functions in the normal distribution. The normal distribution $N(\mu, \sigma^2)$ can be transformed following Myers and Montgomery (1997) to obtain the following equation:

$$f(y;\mu) = \exp\left\{\frac{y\mu - \mu^2/2}{\sigma^2} - \frac{1}{2}\left(y^2/\sigma^2 + \log(2\pi\sigma^2)\right)\right\}. \tag{20}$$

Hence, in the normal distribution, $\theta = \mu$, $h(\phi) = \sigma^2$, and $V(\mu) = 1$. The canonical link function is μ.

Second, we derive the process mean $E_{\mathbf{z}}(\mu_{ijk})$. Considering that the link function is μ and $q''[\eta_0] = 0$, we can derive the following equation:

$$\begin{aligned} E_{\mathbf{z}}(\mu_{ijk}) &\approx q[\eta_0] + \frac{1}{2}q''[\eta_0]Var_{\mathbf{z}}(\eta) \\ &= q[\eta_0]\,. \end{aligned} \tag{21}$$

When the population mean model of Equation (19) is substituted, the estimation model of the process mean is given as:

$$\hat{E}_{\mathbf{z}}(\mu_{ijk}) \approx (7.42 - 1.78C - 0.10D)M. \tag{22}$$

Third, the process variance, $Var(y_{ijk})$, is derived. Notably, $\Psi = 1$ because we use the established link functions and $V(\mu) = 1$ because we use normal distributions. Therefore, the following equation is derived:

$$Var(y_{ijk}) \approx (\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})^T + \phi_{ik}. \tag{23}$$

When the population mean model and dispersion parameter model of Equation (19) are substituted, the estimation model of the process variance is given as:

$$\begin{aligned} \widehat{Var}(y_{ijk}) = {} & (-10.92 + 4.50B - 4.23D)^2\sigma_{\mathbf{z}}^2 + \exp(4.25 - 0.73B \\ & - 0.92C). \end{aligned} \tag{24}$$

Fourth, we define the evaluation function that requires parameters that are arbitrarily determined by the designer based on the purpose and state of the process. The setting used for this analysis is as follows:

$$\begin{gathered} t = 1.0M, \\ M = (4.0, 6.0, 8.0, 10.0, 12.0), \\ \sigma_{\mathbf{z}}^2 = 1.0, \end{gathered} \tag{25}$$

where t denotes the target function, M denotes the standard set of signal factors, and $\sigma_{\mathbf{z}}^2$ denotes the variance of the noise factors. Because an appropriate target function was unknown in the original experiment (Enkawa and Miyakawa, 1992), we define a new one. We define each evaluation function based on the setting and optimize in the range of $-1 \leq x \leq 1$ for each evaluation function. The equation for the evaluation function is omitted.

The details of each design solution are listed in Table 2. SSE is the evaluation function with the normal RSM approach when the $adjusted\,SSE$ is fixed at

$\kappa = 1$. The SSE is minimized using the quasi-Newton method.

As presented in Table 2, the design solution using the $adjusted\ SSE$ is consistent with the conclusions of Enkawa and Miyakawa (1992). The first term of the $adjusted\ SSE$ is diminutive owing to the adjustment parameter. Therefore, we calculate and compare the first term of the SSE using the design solution of the $adjusted\ SSE$. It can be observed that the SSE is smaller, indicating that it derives the design parameters by drawing the process mean closer to the target function. Moreover, because the adjustment parameter is 0.17, the target function is less than the estimated process model. The second term allows for a simple comparison because no adjustment exists in either evaluation function. By comparing the values of the second term, it is evident that the $adjusted\ SSE$ derives solutions that have a more attenuated process variation than the SSE.

Table 2 – Details of design solution

Method	*B*	*C*	*D*	First term	Second term	Evaluation value
$SSE, \kappa = 1.00$	1.00	1.00	0.26	7678.31	348.66	8026.97
$adjusted\ SSE, \kappa = 0.17$ (calculated SSE value)	1.00	1.00	−1.00	0.05 (8083.38)	91.28	91.33 (8174.66)

Next, we visually evaluate the quality of each design solution. We show the relation plot between the level of the signal factor and the response in the obtained design solution in Figure 1. The vertical axis of the graph shows the response y and the horizontal axis shows the level of signal factor M. The target function t is indicated by a solid line; the process mean $E_z(\mu_{ijk})$ is indicated by a dotted line, and the population mean μ_{ijk} of the response at each level of the noise factor Z is indicated by four white points. Furthermore, the level of the signal factor M is plotted in 1.0 increments 4.0 of 12.0. The noise factor Z has four levels $(-1.0, -0.5, 0.5, 1.0)$.

As illustrated in Figure 1, the solution designed using SSE gets closer to the target function by increasing the process variation. Conversely, the solution designed using $adjusted\ SSE$ decreases the process variation while drawing the process mean sufficiently closer to the target function.

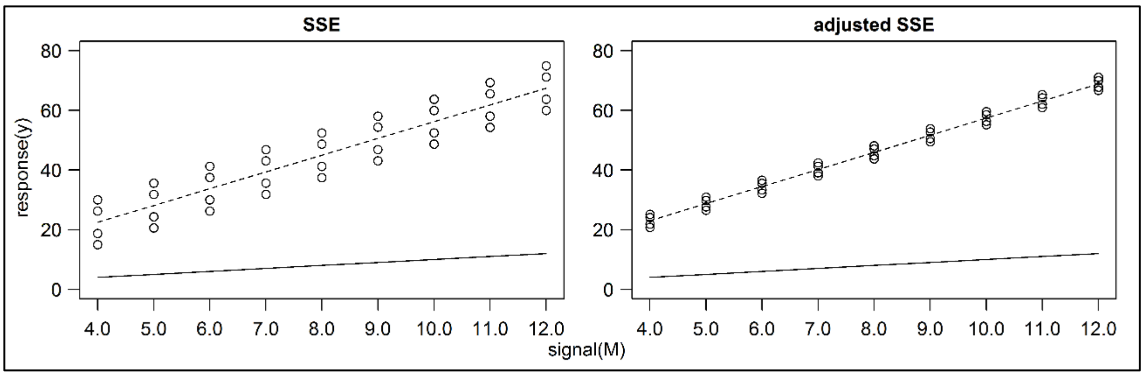

Figure 1 – Relation plots between signal factor levels and responses in each design solution

5 VERIFICATION BY SIMULATION

5.1 Simulation settings

This section describes the simulations performed to verify $adjusted\ SSE$. The differences and variations between the ideal and design solutions in the simulation model are verified as evaluation criteria to clarify whether our solution is robust.

We set the simulation model for response y, as follows:

$$\begin{gathered} y = \varepsilon, \\ \log(\mu) = \eta \\ = 1.5 + 0.5B + (0.25A + 0.5B)M + (1.0A + 1.0C + 1.0D)Z. \end{gathered} \tag{26}$$

The error ε follows a Poisson distribution $P(\lambda)$ and $\lambda = \mu$. Hence, this simulation does not conform to the typical dynamic RPD assumption. The conditional mean and variance are expressed as

$$E\left(y_{ijk}|\mathbf{x}_i, \mathbf{z}_j, \mathbf{m}_k\right) = \mu_{ijk}, \tag{27}$$

$$Var\left(y_{ijk}|\mathbf{x}_i, \mathbf{z}_j, \mathbf{m}_k\right) = \mu_{ijk}. \tag{28}$$

Moreover, our simulation does not assume over- or under-dispersion. Therefore, we use the GLM to estimate the RSM.

The solution $(A, B, C, D) = (1.0, 1.0, -0.5, -0.5)$ is considered ideal because it maximizes the intercept and slope while nullifying the effect of the noise factor. However, extrapolated solutions are not assumed. Substituting the ideal solution into Equation (26) yields the following:

$$\log(\mu) = 2.0 + 0.75M. \tag{29}$$

Therefore, we set the target function t as in

$$\log(t) = 2.0 + 1.0M. \tag{30}$$

The slope of this target function exceeds that in Equation (29). If the design solution increases the process variation, the process mean can be brought closer to the target function. However, this design solution is not robust. Therefore, in this simulation, the average of the design solutions is close to $(A, B, C, D) = (1.0, 1.0, -0.5, -0.5)$; the smaller the variance in the design solutions, the better the solution.

In the framework in GLM, where neither overdispersion nor underdispersion is assumed, the Poisson distribution $P(\mu)$ can be transformed following Myers and Montgomery (1997) to obtain the following equation:

$$f\left(y_{ijk}; \mu_{ijk}\right) = \exp\left\{y_{ijk}\log\left(\mu_{ijk}\right) - \mu_{ijk} - \log\left(y_{ijk}!\right)\right\}. \tag{31}$$

Hence, in the Poisson distribution, $\theta = \log(\mu)$, $h(\phi) = 1$ and $V(\mu) = \mu$. The canonical link function is the $\log(\mu)$.

First, we derive the process mean $E_{\mathbf{z}}(\mu_{ijk})$. Considering that we use a logarithmic link function, $q''[\eta_0] = e^{\eta_0}$ and $E_{\mathbf{z}}(\mu_{ijk})$ can be derived as follows:

$$\begin{aligned} E_{\mathbf{z}}(\mu_{ijk}) &\approx q[\eta_0] + \frac{1}{2} q''[\eta_0] Var_{\mathbf{z}}(\eta) \\ &= e^{\eta_0}\left\{1 + \frac{1}{2}(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega}) Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})^T\right\}. \end{aligned} \tag{32}$$

Second, we derive the process variance, $Var(y_{ijk})$. The established link function is $\log(\mu)$;hence, $\Psi = 1$. Considering that $V(\mu_{ijk}) = \mu_{ijk}$ and $\phi_{ik} = 1$ in the Poisson distribution, the process variance $Var(y_{ijk})$ is given by

$$\begin{aligned} Var(y_{ijk}) &\approx e^{2\eta_0}(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega}) Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})^T \\ &+ e^{\eta_0}\left\{1 + \frac{1}{2}(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega}) Var_{\mathbf{z}}(\mathbf{z}_j)(\boldsymbol{\gamma} + \mathbf{x}_i^T\boldsymbol{\Lambda} + \mathbf{m}_k^T\boldsymbol{\Omega})^T\right\}. \end{aligned} \tag{33}$$

Each parameter used for the design is given in the following equation:

$$\begin{aligned} M &= (1.0, 1.5, 2.0, 2.5, 3.0), \\ \sigma_{\mathbf{z}}^2 &= 1.0. \end{aligned} \tag{34}$$

The equation of the evaluation function is omitted.

The data used for the simulation were generated using the orthogonal array L_{16}. Factors A, B, C, and D are assigned to 1st, 2nd, 4th, and 8th columns, respectively. The noise factor Z and signal factor M are arranged in a one-by-one outer array. The first level of the noise factor is 1, while the second is -1. The first, second, and third levels of the signal factor are 1, 2, and 3, respectively. The number of simulations is 10,000. In the real data analysis, the SSE —a normal RSM approach—is also used for comparison.

5.2 Simulation results

Figure 2 shows the simulation results. The red cross symbol in the box plot represents the ideal solution level, and the blue point is the average of each design solution.

As indicated in Figure 2, the control factors A and B optimized using SSE are closer to the ideal solution, whereas the control factors C and D are significantly far from the ideal solution. In the simulation model, the design solutions other than B are related to the noise factor. These results imply that the process variation increases. Therefore, it can be inferred that the process mean can approach the target function by increasing the process variation without decreasing the influence of the noise factor.

Conversely, the control factor optimized using the $adjusted\ SSE$ is very close to the ideal solution, and the variability of all control factors is diminutive, indicating that the variation is reduced while achieving appropriate input-output relations.

Based on the above, in the simulation of a realistic system, the $adjusted\ SSE$ can obtain a more robust stable design solution against changes in the noise factor and draw the process mean closer to the target function than the SSE.

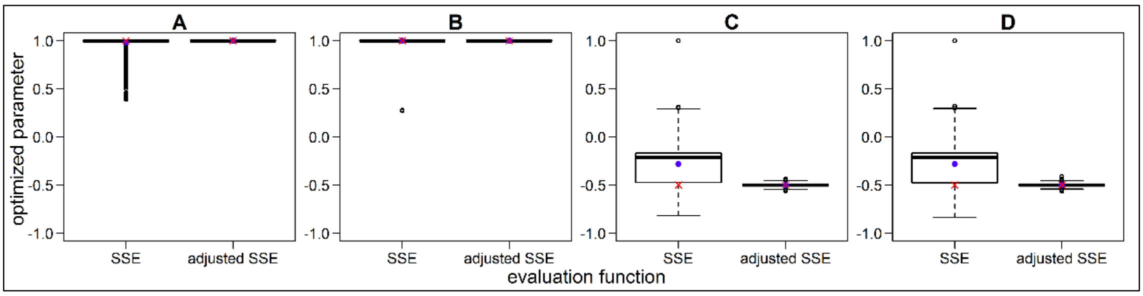

Figure 2 – Boxplot of each design solution derived in simulation

6 CONCLUSION AND FUTURE ISSUES

The dynamic RPD is indispensable for evaluating the robustness of the input-output relation. However, realistic systems often deviate from typical dynamic RPD assumptions. Therefore, this study proposed a novel approach to RPD using RSM based on GLM for dynamic systems based on the method proposed by Myers et al. (2005). Additionally, we demonstrated the effectiveness of our method using dynamic experimental data and simulations.

The actual data analysis reveals that a system design using $adjusted\ SSE$ is found to derive a more robust design solution against the fluctuation of the noise factor than the one using SSE. Moreover, the target function could be evaluated using the adjustment parameter. In the simulation, when the system design using $adjusted\ SSE$ sets a free target function, we could derive a design solution close to the assumed ideal solution. This enables the designer to freely set the target function, design the system with various backgrounds, and subsequently evaluate it.

Future research should analyze the factor(s) in the estimate process model that significantly contributes to the adjustment parameter's variation. We propose introducing the contribution rate as an index to identify these factors. Thus, significant improvements are expected in the process adjustments.

ACKNOWLEDGMENTS

I would like to thank the anonymous referees for their valuable comments. This work was partly supported by JSPS Grants-in-Aid for Scientific Research Grant Number 18K11202 and 21K14372.

REFERENCES

Enkawa, T. and Miyakawa, M., 1992. *SQC Theory and Practice*. Tokyo, Japan: Asakura Publishing.

Kawamura, T. and Takahashi, T., 2013. A Statistical Modeling and Optimization of Dynamic Parameter Design. *Journal of the Japanese Society for Quality Control*, [e-journal] 43(3), pp.364-371. https://doi.org/10.20684/quality.43.3_364.

Kume, S. and Nagata, Y., 2013. Robust Parameter Design for Signal-Response Systems Using Generalized Linear Model. *Asian Network for Quality Congress 2013*, The Swiss Hotel Le Concorde Hotel, Bangkok, 16-18 October 2013.

Lee, Y. and Nelder, J. A., 1998. Generalized Linear Models for the Analysis of Quality-improvement Experiments. *The Canadian Journal of Statistics*, [e-journal] 26(1), pp.95-105. https://doi.org/10.2307/3315676.

Lee, Y. and Nelder, L. A., 2003. Robust Design via Generalized Linear Models. *Journal of Quality Technology*; [e-journal] 35(1), pp.2-12. https://doi.org/10.1080/00224065.2003.11980187.

Mikami, E. and Yano, H., 2004. Studies of a Method of Detecting Thermoresistant Bacteria. *Journal of Quality Engineering Society*, [e-journal] 12(6), pp.37-44. https://doi.org/10.18890/qes.12.6_37.

Myers, R. H. and Montgomery, D. C., 1997. A Tutorial on Generalized Linear Models. *Journal of Quality Technology*, [e-journal] 29(3), pp.274-291. https://doi.org/10.1080/00224065.1997.11979769.

Myers, W. R., Brenneman, W. A. and Myers, R. H., 2005. A Dual-Response Approach to Robust Parameter Design for a Generalized Linear Model. *Journal of Quality Technology*, [e-journal] 37(2), pp.130-138. https://doi.org/10.1080/00224065.2005.11980311.

Nagata, Y., 2009. *Statistical Quality Control - A Step-by-Step Guidebook*. Tokyo, Japan: Asakura Publishing.

Nair, V. N., 1992. Taguchi's Parameter Design: A Panel Discussion. *Technometrics*, [e-journal] 34(2), pp.127-161. https://doi.org/10.2307/1269231.

Nelder, J. A. and Wedderburn, R. W. M., 1972. Generalized Linear Models. *Journal of the Royal Statistical Society*, [e-journal] Series A, 135(3), pp.370-384. https://doi.org/10.2307/2344614.

Smyth, G. K., 1989. Generalized Linear Models with Varying Dispersion. *Journal of the Royal Statistical Society*, Series B, [e-journal] 51(1), pp.47-60. https://doi.org/10.1111/J.2517-6161.1989.TB01747.X.

Smyth, G. K. and Verbyla, A. P., 1999. Adjusted Likelihood Methods for Modelling Dispersion in Generalized Linear Models. *Environmetrics*, [e-journal] 10(6), pp.696-709. https://doi.org/10.1002/(SICI)1099-095X(199911/12)10:6<695::AID-ENV385>3.0.CO;2-M.

Tatebayashi, K., 2004. *Introduction to Taguchi Method*. Tokyo, Japan: JUSE Press.

Yoshino, S., 1995. Optimization of Bean Sprouting Condition by Parameter Design. *Journal of Quality Engineering Society*, [e-journal] 3(2), pp.17-22. https://doi.org/10.18890/qes.3.2_17.

ABOUT AUTHORS

Kosuke Oyama 0000-0002-1195-8172 (K.O.) – Graduate student, Department of Industrial and Management Systems Engineering, Waseda University, Tokyo, Japan, e-mail: whbcbr1000rr@akane.waseda.jp

Masato Ohkubo 0000-0002-1885-6945 (M.O.) – Associate Prof., Department of Business Administration, Toyo University, Tokyo, Japan, e-mail: ohkubo@toyo.jp

Yasushi Nagata 0000-0002-4947-1204 (Y.N.) – Prof., Department of Industrial and Management Systems Engineering, Waseda University, Tokyo, Japan, e-mail: ynagata@waseda.jp

AUTHOR CONTRIBUTIONS

Conceptualization, K.O. and M.O.; Methodology, K.O. and M.O.; Software, K.O.;Validation, K.O.; Formal analysis, K.O.; Investigation, K.O.; Resources, K.O.; Data curation, K.O.; Original draft preparation, K.O.; Review and editing, M.O. and Y.N.; Visualization, K.O.; Supervision, M.O. and Y.N.; Project administration, Y.N.; Funding acquisition, Y.N.

CONFLICTS OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.